TABLE III
THE COMPUTED VALUES OF FREQUENCIES AND INTRINSIC Q FACTORS FOR DIELECTRIC MIC RESONATORS HAVING $\epsilon_r = 36.2(1 - j10^{-4})$, $L = 11.0$ mm, AND $\epsilon_b = \text{Re}(\hat{\epsilon}_r)$

	Permittivity	Dimensions			TE_{011} modes		TE_{012} modes	
	$\hat{\varepsilon}_s$	h mm	a mm	d mm	$f_1^{(20)}$ MHz	$Q_1^{(20)}$	$f_2^{(20)}$ MHz	$Q_2^{(20)}$
1	$3.03 - j1.65 \times 10^{-2}$	2.14	3.995	1.77	8015.2	5520	13528.0	6130
2	$1 - j0$	2.14	3.995	1.77	8056.5	10320	13582.8	10327
3	$3.03 - j1.65 \times 10^{-2}$	4.16	3.015	1.77	8010.4	9970	11637.2	9616

III. COMPUTATIONS RESULTS

Computations of the quasi-TE_{011}-mode resonant frequency of dielectric resonators having the same dimensions and permittivities as in [8] and [9] have been carried out first. The results are presented in Fig. 2(a) and in Table I for the M. Jaworski *et al.* resonator [8] and in Fig. 2(b) and in Table II for that of D. Maystre *et al.* [9]. Fig. 2 shows the convergence of approximate solutions versus the number of basis functions. The convergence of the Rayleigh–Ritz method is worse than Weinstein's and the matching of modal expansions methods. On the other hand, the Rayleigh–Ritz method leads to a simple eigenvalue problem which can be solved faster than the problem of the vanishing determinant which must be solved when those methods are used. For the Rayleigh–Ritz method, basis functions have been chosen in the way described in [14] to get the fastest convergence of the quasi-TE_{011}-mode frequency. Subscripts of the basis functions are shown in Tables I and II. An important feature of the Rayleigh–Ritz method is that it provides upper bounds for true resonant frequencies. Therefore, it is complementary to the Weinstein method, which provides lower bounds for them.

For the proof of this see, e.g., [16]. It can be seen from Table II and Fig. 2(b) that the method of matching of modal expansions also provides lower bound for the quasi-TE_{011}-mode frequency.

Using two complementary methods, one can easily assess the maximum error of calculations of resonant frequencies. It is smaller than half of the difference between the values obtained by these methods.

As the second example, the values of resonant frequencies and intrinsic Q factors of full MIC dielectric resonators, shown in Fig. 1, have been computed. The results are presented in Table III. The first two lines in this table show the influence of the substrate on frequencies and Q values. It is seen that the substrate changes the frequencies less that by 1 percent (note that the dielectric constant of the substrate is low). The influence of substrate losses on the Q values is considerable. For a lossless substrate, the intrinsic Q values are approximately equal to the reciprocals of $\tan\delta$ of the dielectric resonator medium, while for a lossy substrate, the Q values decrease 40 percent. The influence of substrate losses is greater when the h/a ratio is smaller (compare lines 1 and 3 from Table III).

IV. CONCLUSIONS

Accurate values of resonant frequencies and intrinsic Q factors of MIC dielectric resonators could be found by the Rayleigh–Ritz method using electromagnetic fields of a post dielectric resonator as an electrodynamic basis. The method described in this paper allows one to find nonradiating quasi-TE_{0nm} modes. For low-loss resonators, the method provides upper bounds for true resonant frequencies. Application of the method requires the solution of an eigenvalue problem for a complex matrix of not very high order. Upper and lower bounds of resonant frequencies can be assessed if two complementary methods are used for calculations.

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Cross-Coupled Coaxial-Line/Rectangular-Waveguide Junction

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***Abstract*—The analysis of a cross-coupled coaxial-line/rectangular-waveguide junction having dissimilar coaxial lines is presented. An equivalent circuit is deduced for the case where the TE_{10} mode is the only propagating waveguide mode. Experimental/theoretical comparisons are also reported which show the analysis to be very accurate.**

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I. Introduction

Cross-coupled junctions have been used in a wide variety of microwave devices. In the early days, they were used as a means of interconnecting coaxial line and rectangular waveguide. More recently, they have been used in many microwave electronic circuits (e.g., IMPATT diode circuits) and in power combiners.

Whatever the application, the microwave engineer needs to be able to calculate, usually with reasonable accuracy, and often over a significant frequency range, the input impedance at one port of the junction for various load conditions at the other ports.

In spite of the numerous applications of the junction, most designs have been based on empirical knowledge, there having been only a few analyses of the problem reported [1]–[3].

The analyses presented in [1]–[3] related to junctions whose coaxial inputs were identical. In some applications (for example, microwave circuits), the additional freedom of being able to have dissimilar coaxial inputs may provide a useful design variable. Thus, in this paper, the general cross-coupled junction, shown in Fig. 1, is considered by the approach presented in [3].

II. Analysis

Consider the cross-coupled junction shown in Fig. 1. Note that the inner and outer radii of the coaxial apertures at ports 1 and 2 are (a, b_1) and (a, b_2), respectively, and that they have the same inner radii but different outer radii. Ports 1 and 2 are defined such that $b_2 \geqslant b_1$.

This cross-coupled junction may be analyzed by the method presented in [3]. It is assumed that the waveguide has an air dielectric, has perfectly conducting walls, and, in the analysis in this section, that the waveguide is perfectly matched at both ends.

Proceeding as in [3], we represent the junction (for the case where both waveguide ports are matched), by the pair of two-port network equations

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2 \tag{1}$$

where I and V are taken to be [3]

$$I = \frac{2\pi}{\ln(b/a)}\int_{\text{aperture}} H_\theta, \quad V = \int_{\text{aperture}} E_r \tag{2}$$

where E_r and H_θ are the radial component of the electric field, and the θ-component of the magnetic field at the appropriate aperture, and where b in (2) is b_1 or b_2 appropriate to the aperture being considered.

It now remains to find Y_{11}, Y_{22}, and Y_{21} (of course $Y_{12} = Y_{21}$ by reciprocity). Having found these parameters, we can then consider the junction for any loading of the coaxial ports. For example, if port 2 is loaded by Z_{L_2}, then the input admittance seen at port 1, Y_1, is given by

$$Y_1 = Y_{11} - Z_{L_2}\frac{Y_{12}Y_{21}}{1 + Z_{L_2}Y_{22}}. \tag{3}$$

As in [3], the result for Y_{11} may be obtained by considering the junction with port 2 short-circuited (i.e., $V_2 = 0$). Thus, Y_{11} may be shown to be [3], [4] (time dependence assumed $e^{j\omega t}$)

$$Y_{11} = -\frac{2\pi j}{\eta_0 kh \ln^2(b_1/a)} \cdot \left\{ kh\ln(b_1/a)\cot(kh) - D_0^{11} - 2\sum_{m=1}^{\infty} D_m^{11} \right\} \tag{4}$$

where $j = \sqrt{-1}$, $k = 2\pi/\lambda$, η_0 is the intrinsic impedance of free space, and

$$D_m^{11} = \begin{cases} -\dfrac{\pi}{2}\left(Y_0(\bar{q}_m ka)J_0(\bar{q}_m kb_1) - Y_0(\bar{q}_m kb_1)J_0(\bar{q}_m ka)\right) \cdot \left(\dfrac{J_0(\bar{q}_m kb_1)}{J_0(\bar{q}_m ka)} + j\dfrac{J_0(\bar{q}_m kb_1)Y_0(\bar{q}_m ka) - J_0(\bar{q}_m ka)Y_0(\bar{q}_m kb_1)}{J_0(\bar{q}_m ka)S^*(\bar{q}_m ka, \bar{q}_m kd, e/d)}\right)\Big/\bar{q}_m^2, & \dfrac{m\pi}{kh} < 1 \\ \left(K_0(q_m kb_1)I_0(q_m ka) - K_0(q_m ka)I_0(q_m kb_1)\right) \cdot \left(\dfrac{I_0(q_m kb_1)}{I_0(q_m ka)} - \dfrac{I_0(q_m kb_1)K_0(q_m ka) - I_0(q_m ka)K_0(q_m kb_1)}{I_0(q_m ka)S(q_m ka, q_m kd, e/d)}\right)\Big/q_m^2, & \dfrac{m\pi}{kh} > 1 \end{cases}$$

where

$$q_m = \sqrt{\left(\frac{m\pi}{kh}\right)^2 - 1}$$

$$\bar{q}_m = \sqrt{1 - \left(\frac{m\pi}{kh}\right)^2}$$

$$S^*(\bar{q}_m ka, \bar{q}_m kd, e/d) = H_0^{(2)}(\bar{q}_m ka) + J_0(\bar{q}_m ka) \cdot \left\{ \sum_{\substack{n=-\infty \\ \neq 0}}^{+\infty} H_0^{(2)}(2|n|\bar{q}_m kd) - \sum_{n=-\infty}^{+\infty} H_0^{(2)}(2|n + e/d|\bar{q}_m kd) \right\}$$

$$S(q_m ka, q_m kd, e/d) = K_0(q_m ka) + I_0(q_m ka) \cdot \left\{ \sum_{\substack{n=-\infty \\ \neq 0}}^{+\infty} K_0(2|n|q_m kd) - \sum_{n=-\infty}^{+\infty} K_0(2|n + e/d|q_m kd) \right\}$$

and J_0, Y_0, I_0, K_0, and $H_0^{(2)}$ are Bessel functions of the first and second kinds, modified Bessel functions of the first and second kinds, and Hankel functions of the second kind, respectively.

The expression for Y_{22} may be obtained in the same manner, and is the same as the expression for Y_{11} except that b_2 is substituted for b_1.

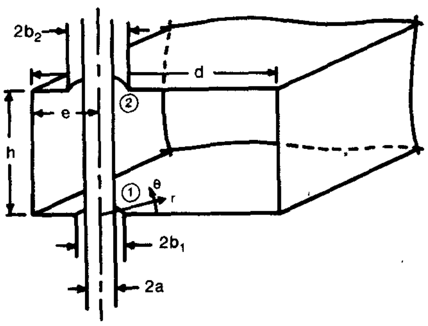

Fig. 1. A sectional view of the cross-coupled coaxial-line/rectangular-waveguide junction.

The result for Y_{21} is given by an expression involving integrals in the plane of aperture 2 (i.e., over the annulus from $r=a$ to $r=b_2$) when port 2 is short-circuited and the junction is driven from port 1. Y_{21} can be obtained in this manner from, and by extension of, the work in [4] and [5]. It should be noted that the magnetic field is given by different expressions in the two regions $a \leqslant r \leqslant b_1$ and $r \geqslant b_1$ ([4, see eq. (9)]). Since it has been assumed that $b_2 \geqslant b_1$, integration over the range a to b_2 needs to be considered in two parts. This is the essential difference between the derivation of Y_{21} for this problem and that for the identical coaxial input cross-coupled junction considered in [3]. A detailed derivation of the result for Y_{21} may be found in [6]. Y_{21} may be shown to be given by [6]

$$Y_{21} = -\frac{2\pi j}{\eta_0 kh \ln(b_1/a)\ln(b_2/a)}\left\{\ln(b_1/a)\frac{kh}{\sin(kh)} - D_0^{21} - 2\sum_{m=1}^{\infty}(-1)^m D_m^{21}\right\} \quad (5)$$

where

$$D_m^{21} = \begin{cases} -\frac{\pi}{2}\left(Y_0(\bar{q}_m ka)J_0(\bar{q}_m kb_1) - Y_0(\bar{q}_m kb_1)J_0(\bar{q}_m ka)\right) \cdot \left(\frac{J_0(\bar{q}_m kb_2)}{J_0(\bar{q}_m ka)} + j\frac{J_0(\bar{q}_m kb_2)Y_0(\bar{q}_m ka) - J_0(\bar{q}_m ka)Y_0(\bar{q}_m kb_2)}{J_0(\bar{q}_m ka)S^*(\bar{q}_m ka, \bar{q}_m kd, e/d)}\right)\Big/ \bar{q}_m^2, & \frac{m\pi}{kh} < 1 \\ \left(K_0(q_m kb_1)I_0(q_m ka) - K_0(q_m ka)I_0(q_m kb_1)\right)\cdot\left(\frac{I_0(q_m kb_2)}{I_0(q_m ka)} - \frac{I_0(q_m kb_2)K_0(q_m ka) - I_0(q_m ka)K_0(q_m kb_2)}{I_0(q_m ka)S(q_m ka, q_m kd, e/d)}\right)\Big/ q_m^2, & \frac{m\pi}{kh} > 1. \end{cases}$$

It should be noted that, as expected, the result (5) for the case $b_2 = b_1$ reduces to the result for Y_{21} given in [3] for the cross-coupled junction having identical coaxial inputs.

III. Equivalent Circuit

The equivalent circuit for the cross-coupled junction shown in Fig. 1 can be deduced by the approach outlined in [3].

Considering the situation where the TE_{10} mode is the only propagating waveguide mode, we can write

$$Y_{11} = \frac{1}{R_1^2 Z_w\left(\frac{1}{2} + jx\right)} + jB_{11}$$

$$Y_{22} = \frac{1}{R_2^2 Z_w\left(\frac{1}{2} + jx\right)} + jB_{22}$$

and

$$Y_{21} = \frac{1}{R_1 R_2 Z_w\left(\frac{1}{2} + jx\right)} + jB_{21}$$

where

$$R_1 = \frac{(2/\pi)\ln(b_1/a)J_0(ka)\sin(\pi e/d)}{J_0(ka)Y_0(kb_1) - J_0(kb_1)Y_0(ka)}$$

$$R_2 = \frac{(2/\pi)\ln(b_2/a)J_0(ka)\sin(\pi e/d)}{J_0(ka)Y_0(kb_2) - J_0(kb_2)Y_0(ka)}$$

$$Z_w = \frac{2kh}{k_{10}d}\eta_0$$

$$k_{10}d = \sqrt{(kd)^2 - \pi^2}$$

$$x = \frac{k_{10}d}{4\pi}\csc^2\left(\frac{\pi e}{d}\right) \cdot \left\{\ln\left(\frac{Ckd}{\pi}\sin\left(\frac{\pi e}{d}\right)\right) - 2\sin^2\left(\frac{\pi e}{d}\right) + 2\sum_{n=2}^{\infty}\sin^2\left(\frac{n\pi e}{d}\right) \cdot \left[\frac{1}{\sqrt{n^2 - (kd/\pi)^2}} - \frac{1}{n}\right] - \frac{\pi}{2}\frac{Y_0(ka)}{J_0(ka)}\right\}$$

$$C = 1.78107\cdots$$

$$B_{11} = -\frac{2\pi}{\eta_0 \ln(b_1/a)}\cot(kh) + \frac{2\pi}{\eta_0 kh \ln^2(b_1/a)} \cdot \left\{2\sum_{m=1}^{\infty} D_m^{11} + \frac{\pi}{2}\frac{J_0(kb_1)}{J_0(ka)} \cdot \left[J_0(ka)Y_0(kb_1) - J_0(kb_1)Y_0(ka)\right]\right\}$$

and

$$B_{21} = -\frac{2\pi}{\eta_0 \ln(b_1/a)}\frac{1}{\sin(kh)} + \frac{2\pi}{\eta_0 kh \ln(b_1/a)\ln(b_2/a)} \cdot \left\{2\sum_{m=1}^{\infty}(-1)^m D_m^{21} + \frac{\pi}{2}\frac{J_0(kb_1)}{J_0(ka)} \cdot \left[J_0(ka)Y_0(kb_2) - J_0(kb_2)Y_0(ka)\right]\right\}$$

while B_{22} is given by an expression the same as that for B_{11} but with b_1 replaced by b_2 and D_m^{11} replaced by D_m^{22}.

Following the procedure outlined in [3], we can isolate the post thickness reactance X_B, where

$$X_B = 2\pi k_{10} d(a/d)^2 \sin^2(\pi e/d) Z_w$$

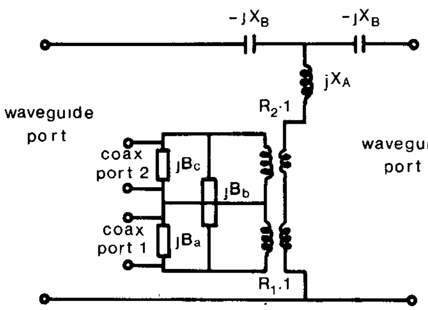

Fig. 2. The equivalent circuit for the junction shown in Fig. 1 for the case where the TE_{10} mode is the only propagating waveguide mode

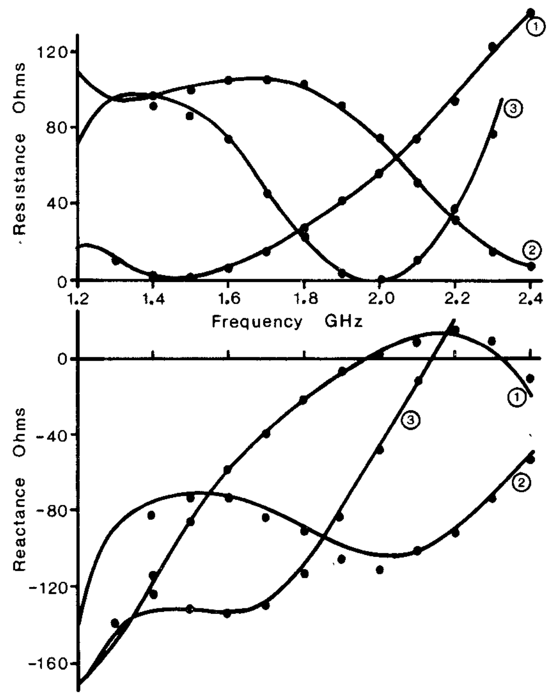

Fig. 3 A comparison of theoretical and experimental results for the input impedance at port 1 of a junction having $a = 0.310$ cm, $b_1 = 0.713$ cm, $b_2 = 1.27$ cm, $h = 5.70$ cm, $d = 12.80$ cm, and $e/d = 0.5$, with both waveguide ports matched, for cases where a short-circuit is applied in the coaxial line at port 2 at distances from port 2 of (1) 3.02 cm, (2) 6.04 cm, and (3) 9.06 cm. ——— theoretical results, · · · experimental results.

and obtain the equivalent circuit for the junction, which is shown in Fig. 2, where $B_a = B_{11} - B_{21}$, $B_b = B_{21}$, $B_c = B_{22} - B_{21}$, and $X_A = xZ_w + X_B/2$.

IV. Comparison of Theory and Experiment

In order to demonstrate the application of the theory presented here, impedance calculations have been made for two situations which have also been investigated experimentally. The numerical results were obtained using the computer programs listed in [6], copies of which are available from the author.

In Fig. 3, theoretical and experimental results are shown for the input impedance at port 1 of a cross-coupled junction having $a = 0.310$ cm, $b_1 = 0.713$ cm, $b_2 = 1.27$ cm, $h = 5.70$ cm, $d = 12.80$ cm, and $e/d = 0.5$ with both waveguide ports matched. Three cases are considered corresponding to short-circuits applied at distances of 3.02 cm, 6.04 cm, and 9.06 cm from port 2 in the coaxial line at that port.

In Fig. 4, theoretical and experimental results are shown for the input impedance for the same situation considered in Fig. 3 except that now $e/d = 0.383$, and only one of the waveguide ports is perfectly matched, the other being short-circuited at a distance of 5.0 cm from the junction.

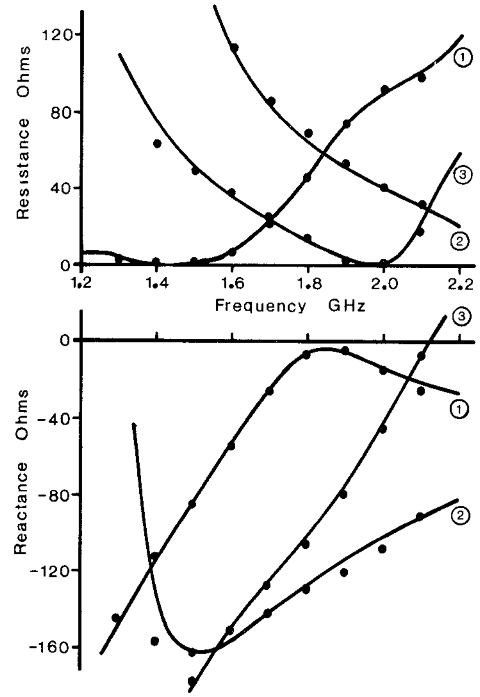

Fig. 4. A comparison of theoretical and experimental results for the input impedance at port 1 of a junction having $a = 0.310$ cm, $b_1 = 0.713$ cm, $b_2 = 1.27$ cm, $h = 5.70$ cm, $d = 12.80$ cm, and $e/d = 0.383$, with one waveguide port perfectly matched and the other short-circuited at a distance of 5.0 cm from the junction, for cases where a short-circuit is applied in the coaxial line at port 2 at distances from port 2 of (1) 3.02 cm, (2) 6.04 cm, and (3) 9.06 cm. ——— theoretical results, · · · experimental results.

Clearly, the agreement between theory and experiment is excellent. This is not unexpected of course, since the theory on which this work is based [3], [4] has previously been shown to yield very accurate results.

V. Conclusion

The analysis of a general cross-coupled coaxial-line/rectangular-waveguide junction having dissimilar coaxial lines has been presented. An equivalent circuit for the junction, applicable to the case where the TE_{10} model is the only propagating waveguide mode, has also been deduced.

Comparison of theoretical and experimental results has shown the analysis to be very accurate.

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